

# 5. Other Cryptographic Constructions Relying on Coding Theory

- Code-Based Digital Signatures
- The Courtois-Finiasz-Sendrier (CFS) Construction
- **Attacks against the CFS Scheme**
- Parallel-CFS
- Stern's Zero-Knowledge Identification Scheme
- An Efficient Provably Secure One-Way Function
- The Fast Syndrome-Based (FSB) Hash Function

# Attacks against a Signature Scheme

As for public-key encryption, there are **two kinds of attacks**.

**Key recovery** attacks:

- try to **recover the secret key** from the public key
- identical to key attacks against McEliece
  - only with different parameters ( $t$  small and  $n$  large)

Nothing different than in McEliece, we will not discuss these here.

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- identical to key attacks against McEliece
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**Forgery** attacks:

- try to **create a valid document-signature pair**
- similar to message attacks against McEliece
- but with no constraint on the document
  - the attacker can **choose the document** freely

# Forgery Attacks

## Counter version

- choose a document  $D$
- pick a counter  $i$
- compute the hash  $h = H(H(D)||i)$
- decode  $h$  as an error of weight  $t$

  $h$  is probably not decodable!

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## Complete decoding version

- choose a document  $D$
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Requires to solve a Syndrome Decoding instance:

- ISD
- GBA

# Forgery Attacks

## Counter version

- choose a document  $D$
- pick **many** counters  $i$
- compute the hash  $h = H(H(D)||i)$
- decode  $h$  as an error of weight  $t$

**⚠ some  $h_i$  are decodable!**

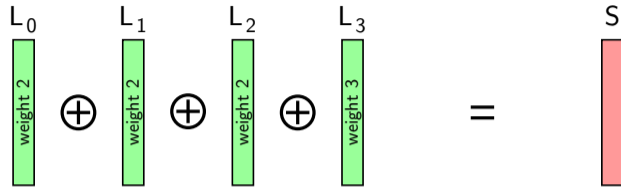
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Requires to solve a Syndrome Decoding instance:

- ISD: best example of **Decoding One Out of Many**
- GBA: build a **list of syndromes**

# Example of Generalized Birthday Attack

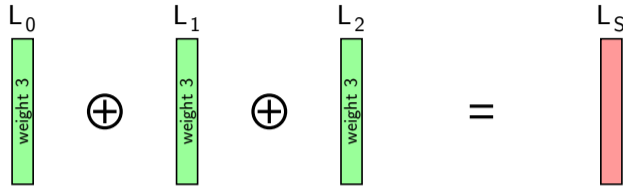


For parameters  $n = 2^{16}$  and  $t = 9$ , the syndromes are 144 bits long.

- the normal GBA setup is to build 4 lists
- it targets a single syndrome  $S$
- lists of  $2^{\frac{144}{3}} = 2^{48}$  elements would find a solution
  - impossible with lists  $L_i$  of weight 2 :  $\binom{2^{16}}{2} = 2^{31}$



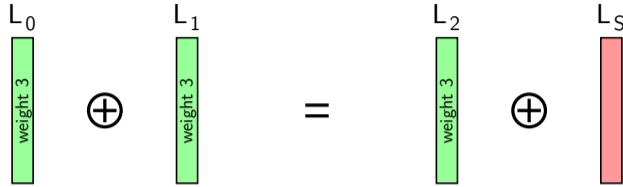
# Example of Generalized Birthday Attack



For parameters  $n = 2^{16}$  and  $t = 9$ , the syndromes are 144 bits long.

- for CFS we target a list  $L_S$  of syndromes
- lists  $L_i$  are a little too small with size  $\binom{2^{16}}{3} = 2^{45.4}$
- $L_S$  has to be made larger to compensate  
→  $L_S$  has size  $2^{60.1}$

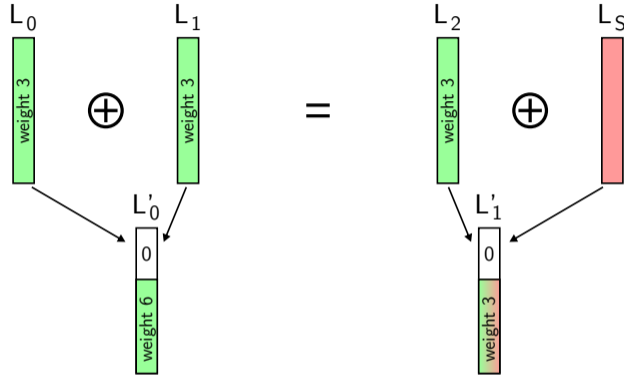
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As usual in GBA, lists are merged by pair:

- $L_0$  with  $L_1$  and  $L_2$  with  $L_S$

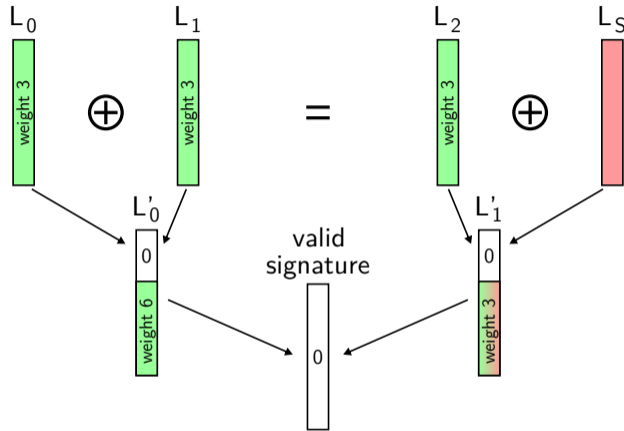
# Example of Generalized Birthday Attack



As usual in GBA, lists are merged by pair:

- $L_0$  with  $L_1$  and  $L_2$  with  $L_S$
- 48 bits of the syndromes are zeroed (96 remain)

# Example of Generalized Birthday Attack



$L'_0$  contains  $\binom{2^{16}}{6} \times 2^{-48} = 2^{38.5}$  elements  
 $L'_1$  contains  $\binom{2^{16}}{3} \times 2^{60.1} \times 2^{-48} = 2^{57.5}$  elements

1 solution is found on average

# Security of the CFS Signature

With the GBA attack, the security of CFS is a little above  $2^{\frac{mt}{3}}$ :

- for  $t = 9$ , a security of  $2^{80}$  requires  $m = 26$
- the public key is then a  $234 \times 2^{26}$  binary matrix
  - its size is over 1 gigabyte!

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There are two choices:

- significantly increase  $t$ 
  - but signature cost is dependent on  $t!$
- or find a way to maintain the security closer to  $2^{\frac{mt}{2}}$

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